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13. ABSTRACT *(Maximum 200 words)*

The work accomplished in this project consists of the following contributions: a) onset-of- failure surface calculations for finitely strained continua with periodic microstructures - which have either voids or rigid inclusions - and which are subjected to arbitrary loading in planc strain (2D), b) failure surfaces for composites under combined axial and shear loading and comparison of the calculations with experimental results carried out at the University of Texas at Austin, c) onset-of-failure surface calculations for finitely strained continua with prismatic microstructures-but arbitrary 2D cross section of the unit cell - which are subjected to arbitrary 3D loading and finally d) development of a consistent criterion for the onset of failure in rate sensitive materials.

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ATTENTION: Dr. L. Lee
Final Report for AFOSR Grant: F49620-99-1-0098

Project title: **INSTABILITY & FAILURE IN DUCTILE SOLIDS WITH REGULAR MICROSTRUCTURES**

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Grant Duration: **Started on 1/1998, ended on 7/2002 (no-cost extension)**

Report Date: **September 1, 2003**

Contents:

1. Background

a) **Objectives:** The objective of this project is to predict failure mechanisms in solids with regular microstructures and to calculate their failure envelopes in macroscopic stress or strain space under arbitrary macroscopic loads and for a wide range of constitutive properties.

b) **Novelty of approach:** The novelty of the present approach is the use of the solid's unit cell, but with boundary conditions that are appropriately modified as to take into account the correct interaction with the neighboring cells (Bloch wave type analysis) to detect the onset of an instability.

c) **Air Force relevance**: The relevance of the project pertains to the design of lighter and stiffer composites and foam type materials by enabling us to propose new unit cell microstructures. Composites, honeycombs and foams are widely used in modern aircraft structures. More particularly our work has been used by Sandia Labs to study some new types of reinforced honeycombs used in shock mitigation.

d) Experimental Collaboration: There is also an important experimental component of this investigation. The theoretical work done at the University of Michigan is closely related to experimental work on fiber-reinforced, graphite-epoxy composites and on rate-sensitive cellular solids under multi-axial loading. The experimental work is carried out by Prof. S. Kyriakides at the University at Texas Austin under a companion AFOSR grant.

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2. Work Accomplished

The work accomplished in this project consists of the following contributions: a) onset-of-failure surface calculations for finitely strained continua with periodic microstructures – which have either voids or rigid inclusions – and which are subjected to arbitrary loading in plane strain (2D), b) failure surfaces for composites under combined axial and shear loading and comparison of the calculations with experimental results carried out at the University of Texas at Austin, c) onset-of-failure surface calculations for finitely strained continua with prismatic microstructures – but arbitrary 2D cross section of the unit cell – which are subjected to arbitrary 3D loading and finally d) development of a consistent criterion for the onset of failure in rate sensitive materials.

2.1/ Onset of failure surfaces in two-phase solids under arbitrary 2D loading

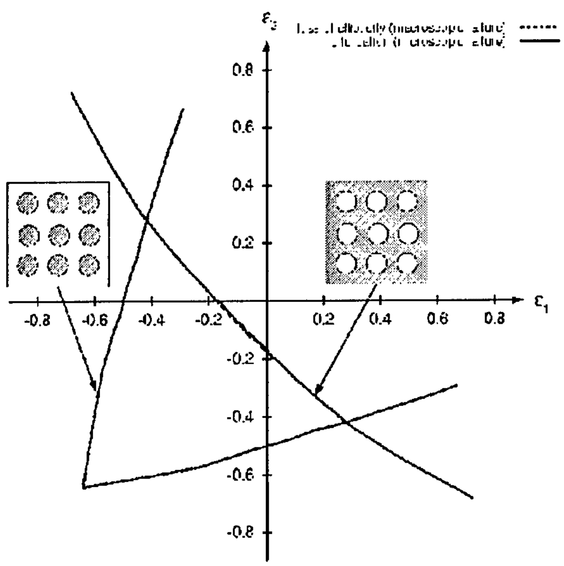


FIGURE 1: *Onset of failure surfaces for solids with voids and with rigid inclusions, under proportional loading in plane strain.*

This part of the work pertains to the failure surfaces – in macroscopic strain space – of periodic fiber reinforced composites and foams under arbitrary plane strain loading. Some representative results are given here, but for more details we refer to the final publication submitted to J.M.P.S.

In each case we calculate both the macroscopic failure (loss of ellipticity of the homogenized moduli) and the microscopic failure (onset of bifurcation away from the periodic principal solution) surfaces. Our objective is to find the influence of the load path and of the microgeometry on the failure surfaces.

The calculations shown in FIGURES 1 and 2 correspond to a hyperelastic material with voids (indicated by white circles on dark background) or with rigid inclusions (indicated by dark circles on a white background). In each case the volume fraction (of the voids or the rigid inclusions) is kept constant at 20%. The macroscopic loading considered is a proportional biaxial straining with respect to the principal axes of the material. The failure surfaces are plotted in macroscopic strain space, where the two axes correspond to the two principal logarithmic strains of the deformation.

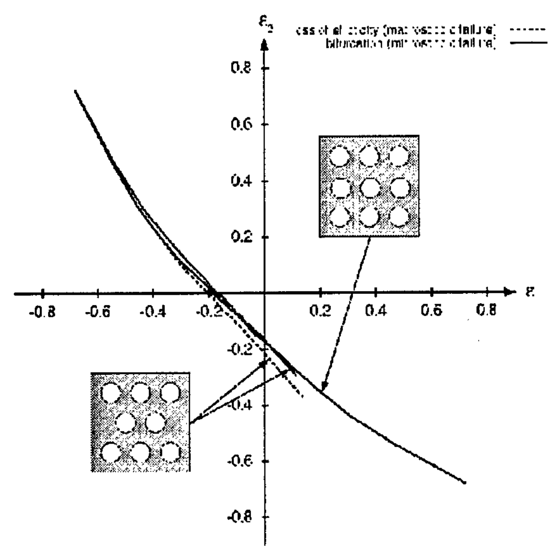


FIGURE 2: *Onset of failure surfaces in plane strain for voided solids with different geometric arrangements but same porosity*

The results in FIG. 1 correspond to square packing of voids and inclusions. Notice that in the case of inclusions, the macroscopic and microscopic failure surfaces coincide, which means that the lowest buckling mode of the solid has a characteristic length much larger than the unit cell size. This is also the case for the foam material, with the exception of the area near balanced biaxial compression loading paths. The importance of the geometric arrangement of voids on the failure surfaces, is shown in FIG. 2, in which we compare the square and diagonal packed foams with the same volume fraction. Notice the significant difference in the failure surface predictions when the microgeometry of the foam is changed.

The calculation of these failure surfaces required the development of a novel computational algorithm, which reduces the final size of the Bloch wave stability matrices (Hermitian) by orders of magnitude. Further work, which compares the onset-of-failure results for the infinite periodic solid to the failure of finite sized specimens containing large number of unit cells are currently underway in a collaboration with the solids research group at LMA at Marseille (see further details in the discussion of transitions section). The work appears in the paper entitled: *Failure Surfaces for Finitely Strained Two-Phase Periodic Solids Under Arbitrary Plane Strains,* which is currently under review.

2.2 Influence of shear on axially compressed composites

This application of the onset of failure surfaces to fiber-reinforced composites, for which recent experimental data by T. Vogler and Kyriakides (IJSS, 1999) on graphite/peek composites were done under AFOSR sponsorship.

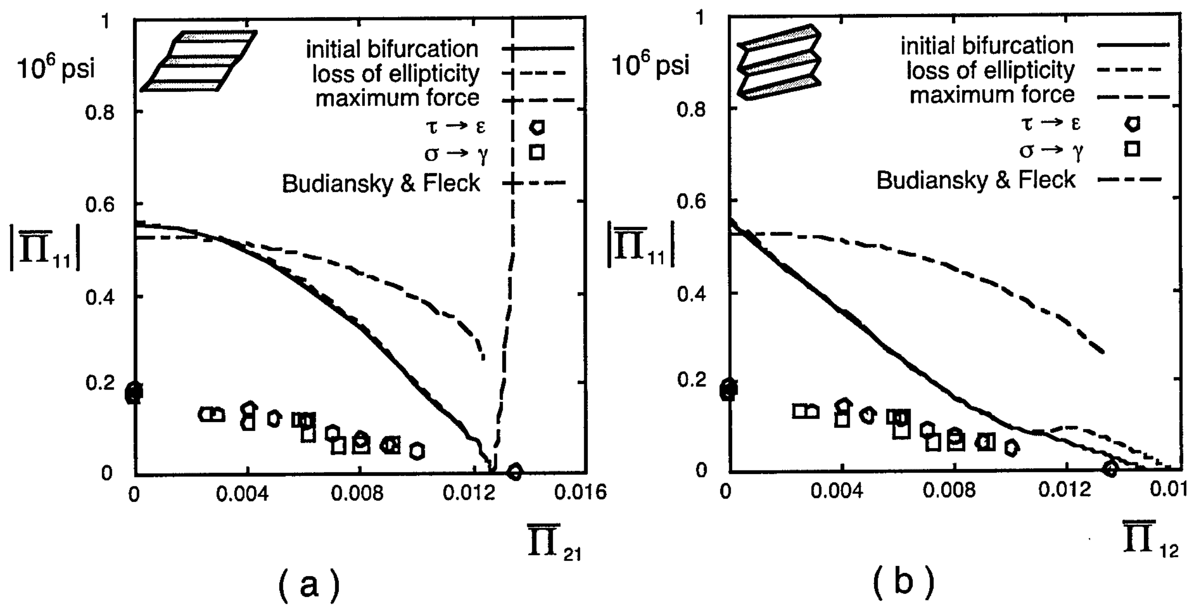


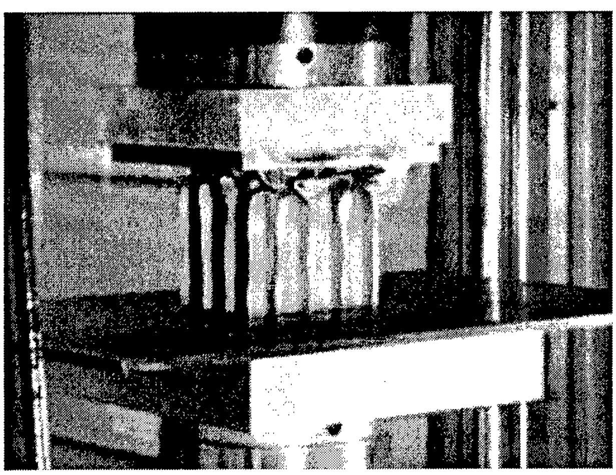
FIGURE 3: *Onset-of-failure surfaces for fiber reinforced AS4-Peek composites under combined axial and shear stresses. Theoretical failure surfaces for infinite perfect solid are compared to structural approximations by Budiansky and Fleck (JMPS, 1994) and to experimental data on actual finite size, imperfect specimens by Vogler and Kyriakides (IJSS 1999). Cases a) and b) correspond to shearing parallel and normal to the fibers.*

The influence of shear on axially compressed composites is an extremely important practical problem, given the fact that it reduces their compressive strength. The novelty of this work is that for the first time an exact – continuum mechanics based – calculation has been carried out for the onset of instability in a composite material, which is perfect, of infinite extent and is subjected to an arbitrary plane strain loading. This way we can separate the influence of imperfections and finite boundaries and find the theoretically maximum loading that can be sustained by the composite. The properties of the composite's phases were measured experimentally and the results from our calculations were compared i) to experimental results for finite sized, imperfect specimens and ii) to approximate structural models that have also been proposed for the perfect composite of infinite extent (the model by Fleck and Budiansky (JMPS 1994) being one of the most popular in the literature has been selected for comparison purposes).

The results plotted in FIG. 3, show the onset of failure surfaces in macroscopic stress space (compressive axial Π_{11} versus shear stress Π_{21} or Π_{12}, depending on the type of loading). As expected, the continuum mechanics calculations (solid line) show that for a given shear stress an infinite perfect composite can sustain larger compressive loads than its imperfect finite-sized counterpart (dots and circles, marking experimental points). It is noteworthy that for large shear stresses the theoretical and experimental calculations are in considerably closer agreement. The increasing discrepancy between the exact continuum mechanics calculations (solid line) and the beam theory structural approximation (dotted line) for larger shear strains is due to the simplifying assumptions used in the structural model. Although the strains involved rather small (a few percent maximum), the way the shearing load is applied does make a difference in the predicted failure surface as seen by comparing FIGURES 3a and 3b, a fact that has never been discussed in the literature. An extensive set of calculations and comparisons for different types of composites and loads, as well as a discussion of the type of the first instability (local or global) is given in the paper: *Onset of Failure in Finitely Strained Layered Composites Subjected to Combined Normal and Shear Loading*, to appear shortly in 2003 in the Journal of the Mechanics and Physics of Solids.

2.3 Continua with prismatic 2D microstructures under 3D loading

Another part of the work consisted on generalizing the method for calculating the onset-of-failure surfaces in 2D periodic solids to the case of prismatic solids in 3D. Fiber reinforced composites have this kind of a microgeometrv and exact (from the continuum mechanics standpoint) 3D calculations are important studyi (impossible



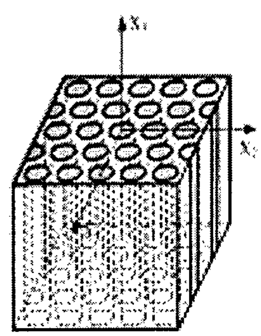


FIGURE 4: *Instability in periodic solids with prismatic microstructures (the unit cell has an arbitrary $x_1 - x_2$ geometry which is independent on x_3). A schematic representation of periodic, prismatic solids is shown on the right, while an experiment showing a compressive buckling mode of such a solid is shown on the right.*

from existing 2D calculations which only take into account volume fraction information).

Three-dimensional Bloch wave calculations for fiber reinforced composites of infinite extent under arbitrary 3D macroscopic loading were performed. The difficulty for the determination of the microscopic failure surface lies in the efficient numerical implementation of the Bloch wave calculations on the continuum unit cell, which requires the scanning of all possible wave numbers in a three-dimensional Fourier space. Due to the invariance of geometry along the axial direction only a 2D mesh is required for solving the 3D problem. We have a special algorithm, which requires the analysis of a considerably reduced problem for each set of wavenumbers, thus minimizing the computational time required for scanning the wave number space. A paper on this work is currently been wrapped-up and will be submitted for publication shortly.

2.4/ Onset of failure surfaces for rate-sensitive solids

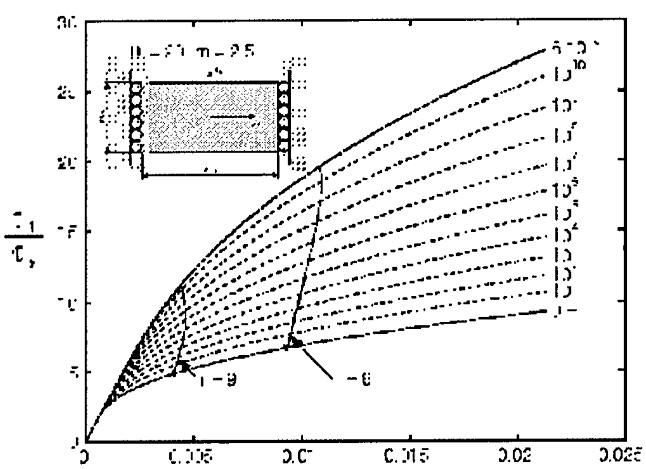


FIGURE 5: *Onset of instability of rate-sensitive bar in plane strain uniaxial compression. Notice the critical stress and strain increase compared with the rate-independent case*

We have developed a consistent criterion that detects the onset of instability in an elastoviscoplastic solid. Our criterion investigates the growth or decay of a perturbation in the principal trajectory of the solid. We have completed the analytical study of stability for a rate-sensitive block under uniaxial tension and compression as a function of its loading rate. A typical result is shown in FIG. 5, which shows the delay in the onset of instability in a compressed elasto-viscoplastic block. A paper describing the solution of this problem and entitled: *On the Stability of Rate-Dependent Solids with Application to the Uniaxial Plane Strain Test* and has been published in 2000 in the Journal of the Mechanics and Physics of Solids. We are currently working on applying this methodology to study the onset of failure in fiber reinforced elastoviscoplastic composites.

3. Personnel Supported

Professor N. Triantafyllidis (summer support)
Doctoral student M. Nestorovic (full research assistantship) Thesis Defended: Oct. 2001
Doctoral student R. Elliott (partial support)

4. Publications in Refereed Journals

"*On the Stability of Rate-Dependent Solids with Application to the Uniaxial Plane Strain Test*," (with: M. Nestorovic), Journal Mech. Physics Solids, 48, 2000, pp. 1476-1491

"*Onset of Failure in Finitely Strained Layered Composites Subjected to Combined Normal and Shear Loading*," (with: M. Nestorovic), To appear in Journal Mech. Physics Solids, 2003.

"*Failure Surfaces for Finitely Strained Two-Phase Periodic Solids Under Arbitrary Plane Strains*," (with: M. Nestorovic and M. Schraad), submitted for publication.

There are two additional publications in the process of being submitted: One with Dr. W. Scherzinger at Sandia Labs and another with Dr. J. C. Michel at LMA in Marseille.

5. Conferences & Invited Presentations

Talks on the research sponsored by AFOSR were presented at the following places;

Feb. 1998	GM Technical Center, Warren, MI
May 1998	Wright-Patterson Labs, Dayton, OH
June 1998	ASME/ASCE/SES Joint Meeting, Gainsville FL
Oct. 1998	Aerospace Dept., University of Texas, Austin, TX
Nov. 1998	ASME Winter Annual Meeting, Anaheim CA
Dec. 1998	Laboratoire de Mechanique & Acoustique, Marseille FRANCE
Jan. 1999	Solids Mechanics Lab., Ecole Polytechnique, Paris, FRANCE
Apr. 1999	ALCOA Tech Center, Alcoa Center, PA
Apr. 1999	GM Technical Center, Warren, MI
May 1999	Euromech Conference in Stability, Cachan FRANCE
Jun. 1999	ASME/ASCE/SES Joint Meeting, VPI, Blacksburg VA
Sep. 1999	AFOSR meeting, WPAFB, Dayton OH
Sep. 1999	Sandia National Labs, Albuquerque, NM
Oct. 1999	Newton Inst. Workshop, Cambridge U.K.
Oct. 1999	SES Annual Meeting, U. Texas, Austin TX
May 2000	Brown University, Providence RI
Aug. 2000	ICTAM Congress, Chicago, IL
Oct. 2000	AFOSR meeting, Columbus, OH
Oct. 2000	Sandia National Labs, Albuquerque, NM
Nov. 2000	ASME Winter Annual Meeting, Orlando, FL
Feb. 2001	Los Alamos National Labs, Los Alamos, NM
Mar. 2001	CALTECH, Pasadena, CA
May 2001	IUTAM symposium, U. Texas, Austin TX
Oct. 2001	AFOSR meeting, Washington DC
Nov. 2001	ASME Winter Annual Meeting, New York, NY
Feb. 2002	University of Houston, Houston TX – Southwest Mechanics Series
Mar. 2002	Northwestern University, Evanston IL
Jun. 2002	US National Congress of Applied Mechanics, VPI, Blacksburg VA

6. Interactions / Transitions

a) Our AFOSR supported work on the failure surfaces for periodic structures has attracted the interest of SANDIA LABS. Of particular interest is the application of our work to reinforced aluminum honeycombs, a cellular material used to absorb shocks and to protect sensitive components during crashes. Researchers at SANDIA have conducted extensive experiments and detailed micromechanical calculations of these solids. However, for large scale modeling of entire structures made of these honeycombs, a detailed micromechanical analysis where each cell is discretized with a fine mesh (necessary to capture the localization phenomena) leads to unrealistically large problems, even with today's supercomputers. Moreover, there is a need for understanding the mechanics of failure of these structures under arbitrary macroscopic loads and find the size of a representative volume element for failure calculations.

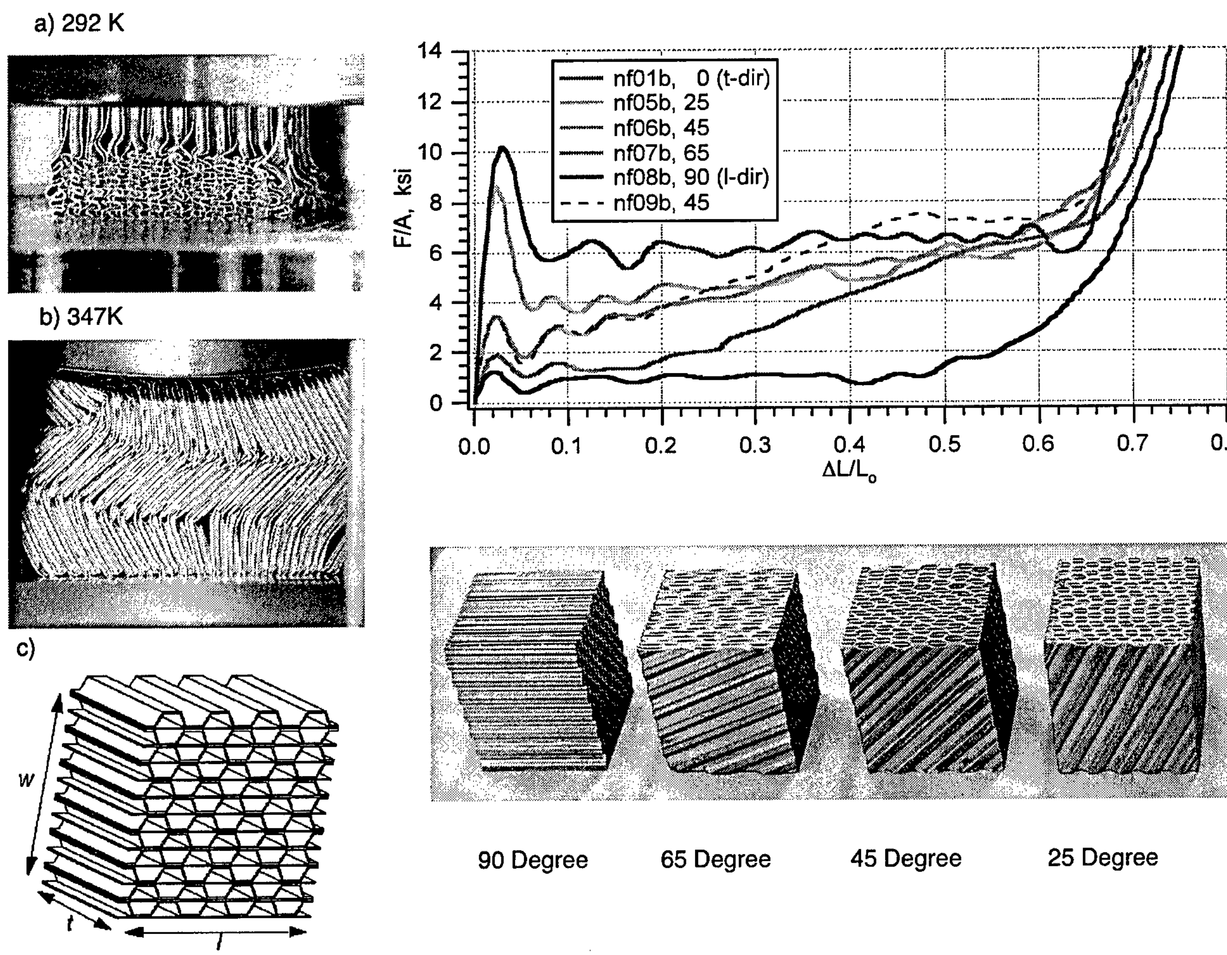


FIGURE 6: *Reinforced Honeycomb whose 3D stability characteristics are investigated at SANDIA. In a) and b) are depicted the failure modes at room and elevated temperatures while c) depicts the Honeycomb's microstructure. The diagram on the right shows the honeycomb's failure at different load orientations and hence the relevance of the failure surface concept developed in our work.*

Staring from August 1999, we have developed a joint research program between SANDIA and U of M to apply our work to these reinforced aluminum solids. The effort at the U of M consists of modeling the onset of failure in the perfect solid under arbitrary 3D loading conditions. The work done in Michigan will be incorporated into codes used by SANDIA to analyze large structures made of the abovementioned reinforced honeycombs.

b) New collaboration has started with scientists at Laboratoire de Mecanique et Acoustique (LMA) at Marseille France. Goal is calculation of failure surfaces for actual two-phase materials (i.e. solids with inclusions or voids which have geometric and material imperfections). Idea is to use fast Fourier-type numerical methodologies developed at LMA, which are ideally suited for large blocks of these materials, which contain a high number of cells. The PI has teamed up with Profs P. Ponte-Castaneda at the University of Pennsylvania and Prof. K Bhattacharya at CATECH on the US plus Ecole Polytechnique Profs. P. Suquet and Dr. J. C. Michel at LMA in France and obtained an NSF-CNRS travel grant to further pursue this collaboration.